

3.3.9 LRFD Stud Shear Connector Design for Straight Girders

For straight girders, LRFD stud shear connector design is similar to LFD shear connector design in positive moment regions where steel beams are made composite with the concrete deck. In both codes, studs are required to be designed for the fatigue and strength limit states. Both codes also require additional studs to be placed near contraflexure points when the negative moment region over piers is non-composite. However, while both have the same basic requirements, the defining equations and procedures often differ. This design guide presents a procedural outline and example calculations for the design of stud shear connectors using LRFD. Part of its focus is comparing and contrasting LRFD with LFD.

The procedures and equations for each aspect of stud shear connector design are given below. The equations and code references shown refer to the AASHTO LRFD Code. Generally, if no commentary is provided, the equations or procedures are either identical or similar to the AASHTO LFD Code.

Provisions for curved steel girder design were merged with the existing provisions for straight girder design in the LRFD Code starting with the 2005 Interims. Many of the shear connector equations in the LRFD Code either refer to curved girders or have aspects embedded in them which are only intended for the design of studs for curved girders. This design guide focuses only on bridges with straight girders. As such, it also serves as a guide for simplifying and separating straight from curved girder LRFD provisions for the design of stud shear connectors.

LRFD Shear Connector Design, Procedure, Equations, and Outline*Determine Dead Load Contraflexure Points*

When finding the dead load contraflexure points, use only the beam and slab (DC1), and superimposed dead loads (DC2). The weight of the future wearing surface (DW) should not be included. The dead load contraflexure points found using the AASHTO LRFD and LFD Specifications should be the same.

Determine Fatigue Loading

There are variances between the fatigue loading provisions for the AASHTO LRFD Code and the LFD Code. The LFD Code uses the standard HS20-44 live loading to determine shear and moment ranges used for fatigue design while LRFD does not use the standard HL-93 loading. Rather, Article 3.6.1.4 specifies a “fatigue truck”. This fatigue truck is similar to the truck portion of the HL-93 load, but has a constant 30 ft. rear axle spacing as opposed to a rear axle spacing which is variable. The LRFD fatigue loading also does not include the distributed lane load included in the HL-93 load.

When using the distribution factors contained in Section 4 of the LRFD Code for fatigue loading, the final distribution factor shall be divided by 1.2 to eliminate the single-lane multiple presence factor which is embedded in the equations (3.6.1.1.2).

For curved roadways on straight bridges, effects of centrifugal forces and superelevation (CE) are included in the LRFD fatigue loading (Table 3.4.1-1, Article 3.6.3). Dynamic load allowance or impact (IM) is also included and taken as 15% of the fatigue truck live load (Table 3.6.2.1-1). This value for impact is reduced from other load cases.

As specified in Table 3.4.1-1, all loads used in LRFD fatigue loading shall be multiplied by a load factor of 0.75.

Check Geometry**Check Stud Dimensions**

The ratio of the height to the diameter of a stud shear connector shall not be less than 4.0 (6.10.10.1.1).

Stud shear connectors should penetrate at least two inches into the concrete deck (6.10.10.1.4). If fillets exceed 6 in., it is IDOT policy to reinforce the fillets to develop the shear studs. See Bridge Manual Section 3.3.9 for further guidance.

Clear cover for shear connectors shall not be less than two inches from the top of slab (6.10.10.1.4).

Calculate Number of Shear Connectors in Cross-Section (6.10.10.1.3)

Stud shear connectors shall not be closer than four stud diameters center-to-center across the top flange of a beam or girder. Article 6.10.10.1.3 requires the clear distance from the edge of the top flange to the edge of the nearest stud connector to be not less than 1 in. It is IDOT policy that the distance from the center of any stud to the edge of a beam shall not be less than 1 ½ in. ¾ in. ϕ studs, which are typically used, and 7/8 in. ϕ studs, which are occasionally used, provide more than the 1 in. clear distance to the edge of the flange required by LRFD. See also Bridge Manual Figure 3.3.9-1.

Find Required Shear Connector Pitch at Tenth-Points of Span

Calculate Pitch for Fatigue Limit State (6.10.10.1.2)

The required pitch, p (in.), of shear connectors shall satisfy:

$$p \leq \frac{nZ_r}{V_{sr}} \quad (\text{Eq. 6.10.10.1.2-1})$$

Where:

n = number of shear connectors in a cross-section

Z_r = fatigue shear resistance of an individual shear connector (kips)

$$= \alpha d^2 \geq \frac{5.5d^2}{2} \quad (\text{Eq. 6.10.10.2-1})$$

Where:

d = stud diameter (in.)

$$\alpha = 34.5 - 4.28 \log N \quad (\text{Eq. 6.10.10.2-2})$$

$$N = \left(365 \frac{\text{days}}{\text{yr.}} \right) (75 \text{ yrs.}) \left(\frac{\text{no. of stress cycles}}{\text{truck}} \right) (\text{ADTT})_{\text{SL}} \quad (\text{Eq. 6.6.1.2.5-2})$$

$$(ADTT)_{SL} = p(ADTT) \quad (\text{Eq. 3.6.1.4.2-1})$$

Where:

p = percentage of truck traffic in a single lane in one direction, taken from Table 3.6.1.4.2-1.

$(ADTT)_{SL}$ is the amount of truck traffic in a single lane in one direction, taken as a reduced percentage of the Average Daily Truck Traffic (ADTT) for multiple lanes of travel in one direction.

ADTT is the amount of truck traffic in one direction of the bridge at 37.5 years. Type, Size, and Location reports usually give ADTT in terms of present day and 20 years into the future. The ADTT at 37.5 years can be extrapolated from this data by assuming that the ADTT will grow at the same rate i.e. follow a straight-line extrapolation using the following formula:

$$ADTT = \left((ADTT_{20} - ADTT_0) \left(\frac{37.5 \text{ years}}{20 \text{ years}} \right) + ADTT_0 \right) (DD)$$

Where:

$ADTT_{20}$ = ADTT at 20 years in the future, given on TSL

$ADTT_0$ = present-day ADTT, given on TSL

DD = directional distribution, given on TSL

So, for example, if a bridge has a directional distribution of 50% / 50%, the ADTT for design should be the total volume divided by two. If the directional distribution of traffic was 70% / 30%, the ADTT for design should be the total volume times 0.7 in order to design for the beam experiencing the higher ADTT. If a bridge is one-directional, the ADTT for design is the full value, as the directional distribution equals one.

The number of stress cycles per truck passage is found in Table 6.6.1.2.5-2.

In the AASHTO LFD Code, at least 2 computations for fatigue strength are required when the ADTT is greater than or equal to 2500. One for $N = 2,000,000$ for multiple-lanes loaded, and one for N “over 2,000,000 cycles”. This second computation assumes single-lane loading with a single-lane distribution factor. The objective of the LRFD Code is to determine a more accurate calculation of the number of fatigue stress cycles in comparison to LFD. This improved fatigue loading determination is used in conjunction with fatigue strength provisions that account for a number of cycles well beyond 2,000,000. The refinement in LRFD fatigue strength provisions from LFD is apparent in the formula used to calculate α which exists in both specifications in different forms. Note, though, that Z_r calculated using LRFD is typically less than for LFD due to the large increase in the number of cycles being considered.

$$\begin{aligned} V_{sr} &= \text{horizontal fatigue shear range per unit length (kip/in.)} \\ &= \sqrt{V_{fat}^2 + F_{fat}^2} \quad (\text{Eq. 6.10.10.1.2-2}) \end{aligned}$$

Where:

$$\begin{aligned} V_{fat} &= \text{longitudinal fatigue shear range per unit length (kip/in.)} \\ &= \frac{V_f Q}{I} \quad (\text{Eq. 6.10.10.1.2-3}) \end{aligned}$$

Where:

- V_f = vertical shear force range under the fatigue load combination specified in Table 3.4.1-1 using the fatigue truck specified in Article 3.6.1.4 (kips).
- Q = first moment of the transformed short-term area of the concrete deck about the neutral axis of the short-term composite section (in.^3).
- I = moment of inertia of the short-term composite section (in.^4)

$$F_{fat} = \text{radial fatigue shear range per unit length (kip/in.)}$$

For straight bridges with skews less than or equal to 45°, F_{fat} may be assumed to be zero throughout the entire bridge for most cases. The commentary of the 2006 interims in LRFD Article 6.10.10.1.2 suggests that radial fatigue analysis should be considered for all structures with skews greater than 20° and discontinuous cross-frames or diaphragms. Typical IDOT detailing practices for cross-frames and diaphragms is not considered discontinuous by the Department. See Section 3.3.5 of the Bridge Manual. As such, radial fatigue shear analysis is unnecessary for most typical bridges with skews less than or equal to 45°.

For straight bridges with skews greater than 45°, the Department recognizes that radial shear should be considered when designing stud shear connectors. Consequently, F_{fat} shall be examined in areas near diaphragms or cross-frames for these bridges, using the following equation:

$$F_{fat} = F_{fat2} = \frac{F_{rc}}{w} \quad (\text{Eq. 6.10.10.1.2-5})$$

Where:

F_{rc} = net range of cross-frame or diaphragm force at the top flange (kips). In lieu of complex analysis, F_{rc} may be assumed to be 25 kips (See LRFD C6.10.10.1.2). The 25 kip load is a conservative estimate of the net range of cross-frame or diaphragm force at the top flange due to the fatigue truck loading.

w = effective length of deck over which F_{rc} is applied. This shall be taken as 24 in. for abutment lateral supports and 48 in. for all other staggered diaphragms or cross-frames.

F_{rc} should only be applied in composite areas within a length of $w/2$ on each side of cross-frames or diaphragms for bridges with skews greater than 45°.

AASHTO LFD Code does not mandate radial analysis for shear connectors design in straight bridges.

Calculate number of required connectors:

The total number of required connectors, n , is calculated using the equation given below. Note that n represents the total number of connectors in the region under consideration, not the number of connectors in a cross-section used in the fatigue limit state check.

$$n = \frac{P}{Q_r} \quad (\text{Eq. 6.10.10.4.1-2})$$

Where:

$$P = \sqrt{P_p^2 + F_p^2} \quad (\text{Eq. 6.10.10.4.2-1})$$

Where:

P_p = total longitudinal shear force in the concrete deck at the point of maximum live load moment (kips), taken as the lesser of:

$$P_{1p} = 0.85f'_c b_s t_s \quad (\text{Eq. 6.10.10.4.2-2})$$

or

$$P_{2p} = F_{yw} D t_w + F_{yt} b_{ft} t_{ft} + F_{yc} b_{fc} t_{fc} \quad (\text{Eq. 6.10.10.4.2-3})$$

Where:

b_{fc} = compression flange width (in.)

b_{ft} = tension flange width (in.)

b_s = effective flange width of composite section (in.)

D = web depth (in.)

t_s = slab thickness (in.)

t_{fc} = compression flange thickness (in.)

t_{ft} = tension flange thickness (in.)

t_w = web thickness (in.)

f'_c = concrete strength (ksi)

F_y = $F_{yw} = F_{yt} = F_{yc}$ = yield strength of steel (ksi)

Note that in wide flange sections and non-hybrid plate girders, Equation 6.10.10.4.2-3 simplifies to:

$P_{2p} = F_y A_{nc}$, where A_{nc} is the total area of steel in the beam

For straight bridges, F_p , or radial shear force in the deck due to live load plus impact, shall be taken as zero (6.10.10.4.2).

Both the LFD and LRFD Codes specify two equations to determine P . The first equation (given above) is for regions from a point of maximum positive moment to a point of zero moment (an abutment or a point of contraflexure) and is used for bridges which are non-composite over piers. The second of these equations is only for use with bridges that are composite in negative moment regions and is beyond the scope of this design guide.

$$Q_r = \phi_{sc} Q_n \quad (\text{Eq. 6.10.10.4.1-1})$$

Where:

$$\phi_{sc} = 0.85 \quad (6.5.4.2)$$

Q_n = nominal resistance of one shear connector (kips)

$$= 0.5 A_{sc} \sqrt{f'_c E_c} \leq A_{sc} F_u \quad (\text{Eq. 6.10.10.4.3-1})$$

A_{sc} = cross-sectional area of one stud shear connector (in.²)

F_u = ultimate strength of stud (ksi) (6.4.4)

E_c = modulus of elasticity of concrete (ksi)

The pitch for the strength limit state is then found using the following equation:

$$p \leq \frac{L}{n} \times \text{no. of connectors in cross-section}$$

Where:

L = length along beam from point of maximum positive live load plus impact to contraflexure point (in.)

As per Article 6.10.10.1.2, the pitch shall not exceed 24 in. nor be less than six stud diameters, regardless of which is the controlling limit state. In regions near points of dead load contraflexure, though, the Department allows longitudinal stud spacing not less than four stud diameters. See the section below for further guidance.

Calculate Number of Additional Connectors at Permanent Load Contraflexure Points

For members that are non-composite in negative moment regions, additional connectors are required. The additional studs shall be placed within a distance of one-third the effective flange width on each side of the point of permanent load contraflexure, and their number, n_{ac} , shall be taken as:

$$n_{ac} = \frac{A_s f_{sr}}{Z_r} \quad (\text{Eq. 6.10.10.3-1})$$

Where:

A_s = total area of longitudinal reinforcement over the interior support within the effective flange width (in.²)

Z_r = fatigue shear resistance of an individual shear connector (kips)

f_{sr} = stress range in the longitudinal reinforcement over the interior support under the fatigue load combination specified in Table 3.4.1-1 with the fatigue load taken as specified in Article 3.6.1.4 (ksi)

$$= \frac{M_r}{S_{top}}$$

Where:

M_r = fatigue truck moment range over the interior support (k-in.)

S_{top} = negative moment section modulus over the interior support (in.³)

The AASHTO LFD Code allows for f_{sr} to be taken as 10 ksi in lieu of more complex computations. This is generally conservative and not mentioned in the LRFD Code. However, the Department allows taking f_{sr} as 10 ksi when using LRFD as a simplification at the discretion of the designer. Note, however, that Z_r in LRFD tends to be significantly smaller than for LFD because cyclic loadings well beyond 2,000,000 are considered. As such, assuming 10 ksi for f_{sr} can produce a number extra shear connectors required which is difficult to detail properly. In addition, even when using a refined analysis to calculate f_{sr} , a significant number of added studs may be still required. As such, the Department allows a longitudinal center-to-center stud spacing as small as 4 stud diameters in regions around dead load contraflexure points for LRFD. For other regions, as stated above, the spacing may only be as small as 6 stud diameters. The minimum allowable spacing in LFD is 4 stud diameters center-to-center.

LRFD Stud Shear Connector Design Example: Two-Span Wide Flange with 30° Skew

Design Stresses

$$f_c = 3.5 \text{ ksi}$$

$$f_y = 60 \text{ ksi (reinforcement)}$$

$$F_y = F_{yw} = F_{yt} = F_{yc} = 50 \text{ ksi (structural steel and stud shear connectors)}$$

$$F_u = 60 \text{ ksi (stud shear connectors)} \quad (6.4.4)$$

$$E_c = 33000K_1 w_c^{1.5} \sqrt{f_c} \quad (\text{Eq. 5.4.2.4-1})$$

Where:

K_1 = aggregate correction factor, normally taken as 1.0

w_c = weight of concrete (kcf). Normal weight of concrete is 0.150 kcf

$$E_c = 33000(1.0) (0.150^{1.5}) \sqrt{3.5} = 3587 \text{ ksi}$$

Bridge Data

Span Lengths:	Two 41 ft. spans
Bridge Roadway Width:	32 ft., 2 lanes of traffic in opposite directions
Beam Size:	W24x68 beams
Stud Shear Connectors:	$\frac{3}{4}$ in. ϕ , 4 in. long
Beam Spacing:	6 ft.
Number of Beams:	6
Slab Depth:	8 in.
ADTT ₀ :	571 trucks
ADTT ₂₀ :	800 trucks
DD:	66.7% / 33.3%

Non-Composite Section Data

A W24x68 beam is used:

$$A_{nc} = 20.1 \text{ in.}^2$$
$$I = 1830 \text{ in.}^4$$
$$d = 23.73 \text{ in.}$$

Composite Section Data

Effective Flange Width = 72 in.

$$Y_b = 24.3 \text{ in. (assuming a } \frac{1}{2} \text{ in. "non-structural" fillet)}$$
$$I = 6260 \text{ in.}^4$$
$$Q = 250 \text{ in.}^3$$

Dead Load Contraflexure Points

33.6 ft. from abutment bearings ($0.82 \times \text{span 1}$, $0.18 \times \text{span 2}$).

Fatigue Load Combination Shears at Tenth-Points of Bridge

This bridge is to be designed non-composite in negative moment regions. Therefore, fatigue load shears need only be found in positive moment regions.

<u>Point</u>	<u>V_{FATIGUE}⁺ (kips)</u>	<u>V_{FATIGUE}⁻ (kips)</u>
0.0	18.9	-1.8
0.1	13.7	-1.7
0.2	11.6	-3.4
0.3	9.6	-5.7
0.4	7.7	-7.8
0.5	5.9	-10.0
0.6	4.3	-12.0
0.7	2.9	-13.7
0.8	0.9	-15.1
0.82	0.4	-15.4

Fatigue Load Moment Range Over Interior Support

$$M_{FATIGUE}^+ - M_{FATIGUE}^- = 120.0 \text{ k-ft.}$$

Longitudinal Deck Slab Reinforcement over Piers

Top of Slab: #5 bars @ 12 in. centers
#6 bars @ 12 in. centers
Bottom of Slab: 6- #5 bars between beams

Check Geometry

Check Stud Dimensions:

Using 4 in. long, $\frac{3}{4}$ in. ϕ studs, the ratio of height to diameter is

$$\frac{h}{d} = \frac{4 \text{ in.}}{0.75 \text{ in.}} = 5.33 > 4 \quad \text{OK} \quad (6.10.10.1.1)$$

Filletts for this bridge do not exceed 2 inches, therefore the stud shear connectors penetrate at least two inches into the slab. The studs are not long enough to extend within two inches of the top of slab (6.10.10.1.4).

Calculate Number of Shear Connectors in Cross-Section: (6.10.10.1.3)

The flange width for a W24x68 is nominally 9 in. Placing the studs a minimum distance of four diameters apart center-to-center with a center-of-stud to edge-of-flange distance of 1 ½ in. allows three studs per row.

Find Required Shear Connector Pitch at Tenth-Points of Span

Calculate Pitch for Fatigue Limit State (6.10.10.1.2)

$$p \leq \frac{nZ_r}{V_{sr}} \quad (\text{Eq. 6.10.10.1.2-1})$$

Where:

$$n = 3 \text{ studs per row}$$

$$Z_r = \alpha d^2 \geq \frac{5.5d^2}{2} \quad (\text{Eq. 6.10.10.2-1})$$

Where:

$$\alpha = 34.5 - 4.28 \log(N) \quad (\text{Eq. 6.10.10.2-2})$$

$$N = \left(365 \frac{\text{days}}{\text{yr.}}\right)(75 \text{ yrs.}) \left(\frac{\text{no. of stress cycles}}{\text{truck}}\right) (\text{ADTT})_{\text{SL}} \quad (\text{Eq. 6.6.1.2.5-2})$$

$$(\text{ADTT})_{\text{SL}} = p(\text{ADTT}) \quad (\text{Eq. 3.6.1.4.2-1})$$

$$\begin{aligned} \text{ADTT} &= \left((\text{ADTT}_{20} - \text{ADTT}_0) \left(\frac{37.5 \text{ years}}{20 \text{ years}} \right) + \text{ADTT}_0 \right) (\text{DD}) \\ &= \left(\left(800 \frac{\text{trucks}}{\text{day}} - 571 \frac{\text{trucks}}{\text{day}} \right) \left(\frac{37.5 \text{ years}}{20 \text{ years}} \right) + 571 \frac{\text{trucks}}{\text{day}} \right) (0.667) \end{aligned}$$

$$= 667 \text{ trucks/day}$$

$$p = 1.0 \text{ for one lane each direction (Table 3.6.1.4.2-1)}$$

$$(ADTT)_{SL} = 1.0 \left(667 \frac{\text{trucks}}{\text{day}} \right) = 667 \frac{\text{trucks}}{\text{day}}$$

$$\frac{\text{no. of stress cycles}}{\text{truck}} = 1.0 \quad (\text{Table 6.6.1.2.5-2})$$

$$N = \left(365 \frac{\text{days}}{\text{yr.}} \right) (75 \text{ yrs.}) \left(\frac{1.0 \text{ stress cycles}}{\text{truck}} \right) \left(667 \frac{\text{trucks}}{\text{day}} \right)$$

$$= 18,300,000 \text{ stress cycles}$$

$$\alpha = 34.5 - 4.28 \log (18,300,000) = 3.42$$

$$Z_r = 3.42(0.75)^2 = 1.92 \text{ kips} \geq \frac{5.5(0.75)^2}{2} = 1.5 \text{ kips}$$

(1.5 kips can be thought of as the allowable for infinite fatigue life)

$$\therefore Z_r = 1.92 \text{ kips}$$

$$V_{sr} = \sqrt{V_{fat}^2 + F_{fat}^2}$$

Where:

$$V_{fat} = \frac{V_f Q}{I}$$

$$Q = 250 \text{ in.}^3$$

$$I = 6260 \text{ in.}^4$$

At points of contraflexure and each tenth point within the positive moment region, V_{fat} is calculated from the shear range V_f as:

<u>Point</u>	<u>V_f (kips)</u>	<u>V_{fat} (k/in.)</u>
0.0	20.7	0.83
0.1	15.4	0.62
0.2	15.0	0.60
0.3	15.3	0.61

0.4	15.5	0.62
0.5	15.9	0.63
0.6	16.3	0.65
0.7	16.6	0.66
0.8	16.0	0.64
0.82	15.8	0.63

$F_{fat} = 0$ kips (diaphragms are considered continuous and skew $\leq 45^\circ$). Therefore, V_{sr}
 $= V_{fat}$.

The resulting maximum pitches for Fatigue Limit State, $\frac{nZ_r}{V_{sr}}$, are:

<u>Point</u>	<u>p (in.)</u>
0.0	6.94
0.1	9.29
0.2	9.60
0.3	9.44
0.4	9.29
0.5	9.14
0.6	8.86
0.7	8.72
0.8	9.00
0.82	9.14

Calculate Pitch for Strength Limit State

(6.10.10.4)

Calculate number of required connectors:

$$n = \frac{P}{Q_r}$$

(Eq. 6.10.10.4.1-2)

Where:

$$P = \sqrt{P_p^2 + F_p^2} \quad (\text{Eq. 6.10.10.4.2-1})$$

Where:

$$P_{1p} = 0.85f_c b_s t_s \quad (\text{Eq. 6.10.10.4.2-2})$$

or

$$P_{2p} = F_y A_{nc} \quad (\text{Simplified Eq. 6.10.10.4.2-3})$$

Where:

$$f_c = 3.5 \text{ ksi}$$

$$b_s = 72 \text{ in.}$$

$$t_s = 8 \text{ in.}$$

$$F_y = 50 \text{ ksi}$$

$$A_{nc} = 20.1 \text{ in.}^2$$

$$P_{1p} = 0.85(3.5 \text{ ksi})(72 \text{ in.})(8 \text{ in.}) = 1713.6 \text{ kips}$$

$$P_{2p} = (50 \text{ ksi})(20.1 \text{ in.}^2) = 1005 \text{ kips}$$

∴ P_{2p} controls

$$F_p = 0 \text{ kips}$$

$$P = 1005 \text{ kips}$$

$$Q_r = \phi_{sc} Q_n \quad (\text{Eq. 6.10.10.4.1-1})$$

Where:

$$\phi_{sc} = 0.85 \quad (6.5.4.2)$$

$$Q_n = 0.5A_{sc} \sqrt{f'_c E_c} \leq A_{sc} F_u \quad (\text{Eq. 6.10.10.4.3-1})$$

Where:

$$\begin{aligned}
 A_{sc} &= \pi(0.375 \text{ in.})^2 = 0.44 \text{ in.}^2 \\
 f'_c &= 3.5 \text{ ksi} \\
 E_c &= 3587 \text{ ksi} \\
 F_u &= 60 \text{ ksi}
 \end{aligned}
 \tag{6.4.4}$$

$$\begin{aligned}
 0.5A_{sc}\sqrt{f'_c E_c} &= 0.5(0.44 \text{ in.}^2)\sqrt{(3.5 \text{ ksi})(3587 \text{ ksi})} = 24.7 \text{ kips} \\
 A_{sc}F_u &= (0.44 \text{ in.}^2)(60 \text{ ksi}) = 26.4 \text{ kips} \\
 \therefore Q_n &= 24.7 \text{ kips}
 \end{aligned}$$

And:

$$Q_r = \phi_{sc} Q_n = 0.85(24.7) = 21.0 \text{ kips}$$

$$n = \frac{1005 \text{ k}}{21.0 \text{ k}} = 47.9 \text{ studs required}$$

Use 48 studs.

Assuming the point of maximum moment occurs at 0.4 of the span length, the required pitch for strength limit state is:

$$p \leq \frac{(0.4 - 0.0)(40 \text{ ft.})\left(12 \frac{\text{in.}}{\text{ft.}}\right)\left(3 \frac{\text{studs}}{\text{row}}\right)}{48 \text{ studs}} = 12.00 \frac{\text{in.}}{\text{row}} \text{ for points 0.0 to 0.4}$$

$$p \leq \frac{(0.82 - 0.4)(40 \text{ ft.})\left(12 \frac{\text{in.}}{\text{ft.}}\right)\left(2 \frac{\text{studs}}{\text{row}}\right)}{48 \text{ studs}} = 12.60 \frac{\text{in.}}{\text{row}} \text{ for points 0.4 to 0.82}$$

The required pitches for fatigue limit state control.

Calculate Number of Additional Connectors at Permanent Load Contraflexure

$$n_{ac} = \frac{A_s f_{sr}}{Z_r} \quad (\text{Eq. 6.10.10.3-1})$$

Where:

$$A_s = A_s^{\text{top of slab}} + A_s^{\text{bot of slab}}$$

$$A_s^{\text{top of slab}} = 6 \text{ ft.} \times \left(0.31 \frac{\text{in.}^2}{\text{ft.}} \right) + 6 \text{ ft.} \times \left(0.44 \frac{\text{in.}^2}{\text{ft.}} \right) = 4.5 \text{ in}^2$$

$$A_s^{\text{bot of slab}} = 6 \times 0.31 \text{ in}^2 = 1.86 \text{ in}^2$$

$$A_s = 4.5 + 1.86 = 6.36 \text{ in}^2$$

$$f_{sr} = \frac{M_r}{S_{\text{top}}}$$

$$M_r = (120.0 \text{ k} - \text{ft.}) \left(\frac{12 \text{ in.}}{\text{ft.}} \right) = 1440.0 \text{ k-in.}$$

$$S_{\text{top}} = \frac{I_{\text{top}}}{y_{\text{top}}}$$

Find \bar{y} from top of slab (assuming a $\frac{1}{2}$ in. fillet):

$$\bar{y} = \frac{A_{nc} y_{nc} + A_s^{\text{top of slab}} y_s^{\text{top of slab}} + A_s^{\text{bot of slab}} y_s^{\text{bot of slab}}}{A_{nc} + A_s^{\text{top of slab}} + A_s^{\text{bot of slab}}}$$

$$A_{nc} = 20.1 \text{ in.}^2$$

$$y_{nc} = 8 \text{ in. slab} + 0.5 \text{ in.} + 0.5(23.73 \text{ in. beam depth}) = 20.37 \text{ in.}$$

$$y_s^{\text{top of slab}} = 2.5 \text{ in. clear} + 0.625 \text{ in. (transverse bar)} + (\approx) 0.375 \text{ in.} = 3.5 \text{ in.}$$

$$y_s^{\text{bot of slab}} = 8 \text{ in. slab} - 1.0 \text{ in. clear} - 0.625 \text{ in. (transverse bar)} - 0.3125 \text{ in.} \\ = 6.06 \text{ in.}$$

$$\therefore \bar{y} = \frac{(20.1 \text{ in.}^2)(20.37 \text{ in.}) + (4.5 \text{ in.}^2)(3.5 \text{ in.}) + (1.86 \text{ in.}^2)(6.06 \text{ in.})}{20.1 \text{ in.}^2 + 4.5 \text{ in.}^2 + 1.86 \text{ in.}^2} = 16.5 \text{ in.}$$

Find I_{top} :

$$I_{\text{top}} = \sum (I + Ad^2) \\ = (I + Ad^2)_{nc} + (Ad^2)_s^{\text{top of slab}} + (Ad^2)_s^{\text{bot of slab}}$$

$$\begin{aligned}
 &= (1830 \text{ in.}^4 + 20.1 \text{ in.}^2(20.37 \text{ in.} - 16.5 \text{ in.})^2) + (4.5 \text{ in.}^2(16.5 \text{ in.} - 3.5 \text{ in.})^2) \\
 &\quad + (1.86 \text{ in.}^2(16.5 \text{ in.} - 6.06 \text{ in.})^2) \\
 &= 3094 \text{ in.}^4
 \end{aligned}$$

Find S_{top} :

$$S_{\text{top}} = \frac{3094 \text{ in.}^4}{16.5 \text{ in.} - 3.5 \text{ in.}} = 238.0 \text{ in.}^3$$

∴

$$f_{\text{sr}} = \frac{1440.0 \text{ k-in.}}{238.0 \text{ in.}^3} = 6.05 \text{ ksi}$$

$$Z_r = \alpha d^2 \geq \frac{5.5d^2}{2} \quad (\text{Eq. 6.10.10.2-1})$$

Where:

$$p = 1.0 \text{ for one lane each direction} \quad (\text{Table 3.6.1.4.2-1})$$

$$\text{ADTT} = 667 \frac{\text{trucks}}{\text{day}}$$

$$(\text{ADTT})_{\text{SL}} = 1.0 \left(667 \frac{\text{trucks}}{\text{day}} \right) = 667 \frac{\text{trucks}}{\text{day}}$$

$$\frac{\text{no. of stress cycles}}{\text{truck}} = 1.5 \quad (\text{Table 6.6.1.2.5-2})$$

$$\begin{aligned}
 N &= \left(365 \frac{\text{days}}{\text{yr.}} \right) (75 \text{ yrs.}) \left(\frac{1.5 \text{ stress cycles}}{\text{truck}} \right) \left(667 \frac{\text{trucks}}{\text{day}} \right) \\
 &= 27,400,000 \text{ stress cycles}
 \end{aligned}$$

$$\alpha = 34.5 - 4.28 \log (27,400,000) = 2.67$$

$$Z_r = 2.67(0.75)^2 = 1.5 \text{ kips} \geq \frac{5.5(0.75)^2}{2} = 1.5 \text{ kips}$$

$$\therefore Z_r = 1.5 \text{ kips}$$

And:

$$n_{ac} = \frac{(6.36 \text{ in.}^2)(6.05 \text{ ksi})}{(1.5 \text{ k})} = 25.7 \quad \text{Use 27 additional studs.}$$

Add 9 additional rows of studs around each contraflexure point. These studs shall be placed within one-third of the effective flange width, or 24 inches, on each side of the contraflexure point.

Design Summary

From the abutment to ± 24 in. past the point of contraflexure the following spacings with 3 studs per row will meet the requirements calculated above:

7 spaces at 7 in., 38 spaces at 8 $\frac{3}{4}$ in., and 15 spaces at 3 in.